2017

AP Statistics

Scoring Guidelines

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Question 1

Intent of Question

The primary goals of this question were to assess a student's ability to (1) explain statistical terms used when describing the relationship between two variables; (2) interpret the slope of a linear regression equation; and (3) calculate a value of y when given a regression equation, a value of x, and a residual.

Solution

Part (a):

In the context of a scatterplot in which y represents weight and x represents length, the following are defined.

A positive relationship means that wolves with higher values of length also tend to have higher weights.

A linear relationship means that as length increases by one meter, weight tends to change by a constant amount, on average.

A strong relationship means that the data points fall close to a line (or curve).

Part (b):

The slope of 35.02 indicates that two wolves that differ by one meter in length are predicted to differ by 35.02 kilograms in weight, with the longer wolf having the greater weight.

Part (c):

In general, a residual is equal to actual weight minus predicted weight, or equivalently,

$$\text{actual weight} = \text{predicted weight} + \text{residual}.$$

For the wolf with length 1.4 meters and residual of -9.67, the predicted weight is

$$-16.46 + 35.02(1.4) = 32.568 \text{ kilograms}.$$

Therefore, the actual weight of the wolf is $32.568 + (-9.67) = 22.898$ kilograms.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes the following four components:

1. A reasonable definition of positive
2. A reasonable definition of linear
3. A reasonable definition of strong
4. At least one definition in context

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Question 1 (continued)

Partially correct (P) if the response includes only three of the four components.

Incorrect (I) if the response does not meet the conditions for E or P.

Notes:

- The description of a positive relationship should clearly indicate that relatively low values of one variable tend to appear with relatively low values of the other variable, and relatively high values of the first variable tend to appear with relatively high values of the other variable.
    - Examples of acceptable responses:
        - As length increases, so does weight.
        - Longer wolves weigh more.
        - The points on the graph go up as you move from left to right.
    - Examples of unacceptable responses:
        - As length goes up, weight changes.
        - Both length and weight get bigger.
        - The correlation is greater than 0.
- The description of a linear relationship can take one of two approaches: the data pattern (data points exhibit the pattern of a line in the graph) or the constant rate of change (as the explanatory variable changes, the response variable exhibits a constant rate of change).
    - Examples of acceptable responses:
        - The points generally follow a straight line.
        - The relationship between x and y is straight.
        - Length and width have a constant slope.
    - Examples of unacceptable responses:
        - The points all line up.
        - You can draw a straight line through the points.
        - There is a positive correlation.
        - Every increase in x yields a 35.02 increase in y.
- The description of strong should indicate how close points are to a line.
    - Examples of acceptable responses:
        - Observed values are close to predicted values.
        - Deviations from the least-squares regression line are small.
        - The correlation coefficient is close to 1.
    - Examples of unacceptable responses:
        - All the points are close together.
        - The scatterplots are clustered together.
        - There is a high positive correlation.
- Context can be shown by referring to length and weight or by using meters and kilograms.
- Sketches and graphs can be used to help clarify definitions, but a sketch alone cannot satisfy a definition component.

Part (b) is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. The correct value of 35.02 for the slope.
2. An interpretation that includes an increase of a specified amount of weight for each unit increase in length.
3. An indication that the relationship is not exact by using words such as "on average" or "predicted weight."

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Question 1 (continued)

Partially correct (P) if the response includes only two of the three components.

Note: If the response identifies the slope as -16.46 (the intercept value), the second component is satisfied only if the response states that for each one-meter increase in length there is a *decrease* in predicted or average weight of 16.46 kilograms.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (c) is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. A correct computation for the predicted value 32.568 kilograms.
2. A correct computation for the actual weight 22.9 kilograms using the given residual and the predicted value.

Partially correct (P) if the response provides a correct computation for the predicted value but is not able to complete the correct calculation of the actual weight, including if the residual is defined in the wrong direction as (predicted weight) – (actual weight) to give an answer of 42.24 kilograms;

OR

if the response provides an incorrect value for the predicted weight, but then uses that value correctly to determine the actual weight as (predicted weight) + residual = (predicted weight) $+(-9.67)$;

OR

if the response provides a correct answer for the actual weight but does not give sufficient information to determine how it was calculated.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- The expression $-16.46 \quad 35.02(1.4)$ is enough to satisfy the first component.
- The equation $-16.46 + 35.02(1.4) - 9.67 = 22.9$ satisfies both components.
- Arithmetic mistakes are overlooked if they do not lead to an unreasonable answer (such as a negative value). For example, $32.568 + (-9.67) = 21.9$ satisfies the second component.

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Question 1 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

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Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) construct and interpret a confidence interval for a population proportion and (2) use a confidence interval for a proportion to find a confidence interval for a dollar amount that can be calculated using that proportion.

Solution

Part (a):

Step 1: Identify the appropriate confidence interval by name or formula and check appropriate conditions.

The appropriate procedure is a one-sample z-interval for a population proportion p. In this case, the population is all customers of the restaurant who ask for a water cup, and p is the proportion of that population who will fill the cup with a soft drink. The appropriate formula is $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

Conditions:

1. Random sample
2. Large sample (number of successes $n\hat{p} \geq 10$ and number of failures $n(1-\hat{p}) \geq 10$)

For condition 1 the stem of the problem states that a random sample of customers who asked for a water cup was used.

For condition 2 the number of successes (filled cup with soft drink) is 23 and the number of failures is 57, both of which are greater than 10.

Step 2: Correct mechanics

The sample proportion is $\hat{p} = \frac{23}{80} = 0.2875$. The confidence interval is

$$0.2875 \pm 1.96\sqrt{\frac{0.2875(1 \quad 0.2875)}{80}}$$
$$= 0.2875 \pm 1.96(0.0506) \qquad \text{or } 0.1883 \text{ to } 0.3867.$$
$$= 0.2875 \pm 0.0992$$

Step 3: Interpretation

We can be 95 percent confident that in the population of all customers of the restaurant who ask for a water cup, the proportion who will fill it with a soft drink is between 0.1883 and 0.3867.

Part (b):

Using the confidence interval in part (a), a 95 percent interval estimate for the number of customers in June who asked for a water cup but then filled it with a soft drink is $3{,}000 \times 0.1883$ to $3{,}000 \times 0.3857$, or 565 to 1,160. At a cost of \$0.25 per customer, a 95 percent interval estimate for the cost to the restaurant in June is \$141.25 to \$290.00.

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Question 2 (continued)

Scoring

This question is scored in four sections. Section 1 consists of step 1 in part (a), section 2 consists of step 2 in part (a), section 3 consists of step 3 in part (a), and section 4 consists of part (b). Each section is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the one-sample z-interval for a proportion is identified (either by name or formula) *AND* both conditions (random sampling and large sample) are adequately addressed.

Partially correct (P) if the response identifies the correct procedure *BUT* adequately addresses only one of the two required conditions;
OR
if the response does not identify the correct procedure *BUT* adequately addresses both required conditions.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Stating the large sample condition without verifying it is not sufficient. The response must use specific numerical values to adequately address the condition.
- If the response includes additional inappropriate conditions, such as $n \geq 30$ or requiring a normal population, then the response earns at most P for section 1.
- Stating and checking a condition about the size of the sample relative to the size of the population is appropriate but not required.
- Any statement of hypotheses, description of the population, or definition of the parameter should be considered extraneous. However, if such statements are included and incorrect, they are considered as poor communication in terms of holistic scoring.

Section 2 is scored as follows:

Essentially correct (E) if the response gives the correct 95 percent confidence interval. Supporting work is not required, but if included, it must be correct.

Partially correct (P) if the response gives a correct confidence interval with incorrect (but appropriate) supporting work shown;
OR
if the response gives an incorrect but reasonable confidence interval with appropriate supporting work shown — for instance, if a value other than 1.96 is used for the critical value or a value other than $\frac{23}{80}$ is used for $\hat{p}$.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Appropriate supporting working must have the form:
  proportion $\pm$ (critical value)(SE/SD of proportion).

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Question 2 (continued)

- A confidence interval that has both endpoints outside the interval from 0 to 1 is considered unreasonable.

Section 3 is scored as follows:

Essentially correct (E) if the response provides an appropriate interpretation of the interval that includes the following three components:

1. Conveys inference about a population proportion
2. Demonstrates a clear understanding that the parameter is the proportion of the water cup population that fills the cup with a soft drink
3. Mentions 95 percent confidence and interprets it correctly using words such as "We can be 95 percent confident" or "With 95 percent confidence"

Partially correct (P) if the response provides an appropriate interpretation of the interval that includes the first component *AND* only one of the other two components;
OR
if the response provides a correct interpretation of the confidence *level* in context without interpreting the specific interval.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Clear indication of an inference to the random sample of 80 customers rather than to the population does not satisfy the first component and is scored I.
- Stating values that are unrealistic as proportions or percentages (including blanks) in the interpretation lowers the score one level (from E to P, or from P to I).
- When both the interpretation and the level of the interval are given, only the interpretation is scored. If the interpretation of the confidence level is incorrect, it is considered as poor communication in terms of holistic scoring.
- Any interpretation that implies the interval has a 95 percent chance (or possibility or probability) of capturing the population proportion is scored I.

Section 4 is scored as follows:

Essentially correct (E) if the response gives a correct interval estimate for the cost to the restaurant *AND* shows enough work to indicate how the interval was found.

Partially correct (P) if the response gives the correct interval estimate for the *number* of customers (565 to 1,160) who would fill the water cup with soda, including showing work, but does not multiply by \$0.25 to find the interval for the cost;
OR
if the response gives the correct interval estimate for the *expected cost* to the restaurant for an individual who asked for a water cup (\$0.05 to \$0.10), including showing work, but does not multiply by 3,000 to find the interval for the total cost;
OR
if the response makes a reasonable attempt to find the endpoints of the interval as $(0.1883)(3{,}000)(0.25)$ and $(0.3867)(3{,}000)(0.25)$, but makes an error such as using the sample size of 80 instead of the population size of 3,000;

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Question 2 (continued)

OR
if the response gives the correct interval without showing how it was found.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- The response in section 4 earns an E if it follows correctly from an incorrect interval found in part (a), even if the interval is unreasonable.
- Units ($) are not required to earn an E.

Each essentially correct (E) section counts as 1 point, and a partially correct (P) section counts as ½ point.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and communication.

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Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate a probability from a normal distribution; (2) calculate a weighted probability from two individual probabilities; and (3) calculate a conditional probability for dependent events when individual and joint probabilities are provided.

Solution

Part (a):

Let X denote the diameter of a randomly selected melon from Distributor J. X has an approximately normal distribution with mean 133 mm and standard deviation 5 mm.

The z-score for a diameter of 137 mm is $z = \frac{137 - 133}{5} = \frac{4}{5} = 0.8$.

Therefore, $P(X > 137) = P(Z > 0.8) = 1 - 0.7881 = 0.2119$.

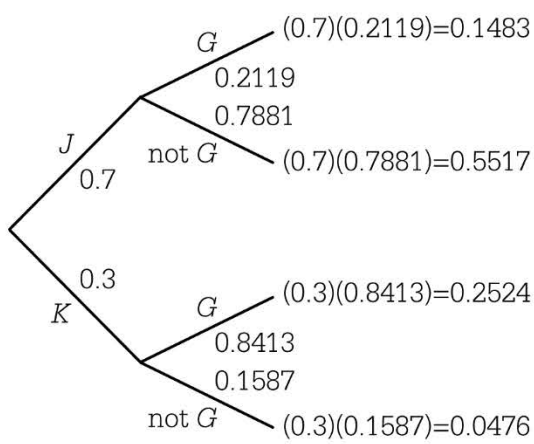
Part (b):

Define events:
J: melon is from Distributor J
K: melon is from Distributor K
G: melon diameter is greater than 137 mm

For a randomly selected melon from the grocery store,

$$\begin{aligned} P(G) &= P(G \mid J) \times P(J) + P(G \mid K) \times P(K) \\ &= (0.2119)(0.7) + (0.8413)(0.3) \\ &= 0.1483 + 0.2524 \\ &= 0.4007 \end{aligned}$$

OR



From the tree diagram, $P(G) = P(G \text{ and } J) + P(G \text{ and } K) = 0.1483 + 0.2524 = 0.4007$.

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Question 3 (continued)

Part (c):

Using the events defined in part (b), the requested probability is

$$P(J \mid G) = \frac{P(J \text{ and } G)}{P(G)} = \frac{P(G \mid J)P(J)}{P(G)} = \frac{(0.2119)(0.7)}{0.4007} = \frac{0.1483}{0.4007} = 0.3701.$$

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response provides the following three components:

1. Normality and parameters: Indicates use of a normal (or approximately normal) distribution and clearly identifies the correct parameter values. (Showing correct components in a z-score, labeling the mean and standard deviation in a normalcdf calculator statement, or drawing a normal curve with sufficient values marked to indicate the mean and standard deviation are all sufficient methods for identifying the correct parameter values.)
2. Boundary and direction: Uses the correct boundary value of $x = 137$ or $z = 0.8$ and the correct direction. (Showing correct boundary and direction in a probability statement using "greater than," labeling the lower limit and upper limit in a normalcdf calculator statement, drawing a normal curve with 137 labeled and an indication that the area of interest is to the right of 137, or a conclusion in words using "greater than.")
3. Probability: Reports the correct normal probability consistent with the response's setup described in components 1 and 2.

Partially correct (P) if the response correctly provides only two of the three components.

Incorrect (I) if the response gives the answer with no work shown or otherwise does not satisfy the criteria for E or P.

Notes:

- An inconsistency in calculations lowers the score for part (a) by one level (from E to P, or from P to I). For instance, if the response states the requested probability as $P(X > 137)$, but actually computes $P(X < 137)$.
- An error in statistical notation, such as using s instead of σ for the standard deviation or $\bar{x}$ instead of μ for the mean, does not satisfy component 1.
- A correct direction can be obtained by using a left direction and an upper bound of 137 and then subtracting the value of the cumulative probability from 1. However, simply showing the calculation $1 - 0.7881 = 0.2119$ does NOT give a correct direction.
- Consider any steps of a hypothesis test as extraneous work.
- Standard notations such as N(133,5) or N(133,25) satisfy the first component.
- A sketch of the normal curve with 133 and 138 in the appropriate positions satisfies the first component.
- Use of a value other than 137 does not satisfy the second component.
- If the only error in part (a) is the reversal of the numerator for the z-score $(133 - 137)$, the response is scored as P.

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Question 3 (continued)

- For calculator notation:
  - ncdf is sufficient to indicate use of the normal distribution.
  - Stating the lower and upper bounds is sufficient for identifying boundary and direction.
  - Minimum notation, such as "u" for upper, is sufficient for identifying boundary.
  - The minimum value for an upper bound must be at least 4 standard deviations above the mean.

Part (b) is scored as follows:

Essentially correct (E) if the probability is computed correctly, *AND* work is shown that includes correct numerical values using a formula, end results from a tree diagram, or some other appropriate method.

Partially correct (P) if the response provides a correct strategy for finding the probability, such as a formula or tree diagram, but uses one or more incorrect values or does not use the end result from the tree;
OR
if the response gives the correct probability but not enough work is shown to determine how it was found.

Incorrect if the response does not meet the criteria for E or P.

Notes:
- A response that uses 0.3 for J and 0.7 for K earns a P if no other mistakes are made.
- Tree diagrams:
  - If used, the multiplication is implied, but the addition must be indicated.
  - If not used, both the multiplication and the addition must be indicated.
- The shown work might be found in part (c).
- Probabilities can be shown in a table. Addition is implied if "Total" is labeled, as shown.

Intersection	G	Not G	Total
J	0.1483	0.5517	0.7
K	0.2524	0.0576	0.3
Total	0.4007	0.5993	

Part (c) is scored as follows:

Essentially correct (E) if the probability is computed correctly *AND* work is shown that illustrates how the probability was found.

Partially correct (P) if the response provides a strategy for finding the probability for dependent variables, such as the appropriate formula or end results from the correct tree diagram, but uses one or more incorrect values or does not use the end result from the tree;
OR
if the response gives the correct probability but not enough work is shown to determine how it was found.

Incorrect (I) if the response does not meet the criteria for E or P.

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Question 3 (continued)

Notes:

- Part (c) earns an E if a correct answer follows from incorrect values used from part (a) and/or part (b), provided all values are between 0 and 1, inclusive.
- A transcription error in a response can be ignored if correct work is shown.
- For any part, if the resulting probability or part of the calculation of the probability uses a value that is not between 0 and 1, inclusive, the score is lowered by one level (from E to P, or from P to I).
- In any part, unsupported answers are scored as I.

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

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Question 4

Intent of Question

The primary goals of this question were to assess a student's ability to use boxplots to (1) compare multiple sets of data; (2) identify which set of data is most likely to have produced a particular summary value; and (3) determine which variable is most useful for classifying a new observation.

Solution

Part (a):

The median value for the percent of chemical Z in the pottery pieces is similar for all three sites, at about 7 percent. The ranges for the percent of chemical Z are much different for the three sites, with the smallest range of about 2 percent (from 6 percent to 8 percent) at Site II, a range of about 6 percent (from about 4 percent to 10 percent) at Site I, and the largest range of about 8 percent (from about 3 percent to 11 percent) at Site III.

Part (b):

(i) The piece most likely originated at Site III. Although values outside of the range of data observed in the samples would be possible, using the available data results in approximate minimum and maximum sums of the percents for the three chemicals as shown in the table below. Site III is the only site in which 20.5 falls between the sums of the minimum and maximum values.

	Site I		Site II		Site III	
Chemical	Min	Max	Min	Max	Min	Max
X	6	8	5	7	5	7.5
Y	11	15	1.9	4	6	8
Z	4	10	6	8	3	11
Sum	21	33	12.9	19	14	26.5

(ii) Chemical Y would be most useful, because the distribution of the percentages of total weights at the three sites do not overlap. The distributions of chemicals X and Z have substantial overlap.

Scoring

This question is scored in three sections. Section 1 consists of part (a), section 2 consists of part (b-i), and section 3 consists of part (b-ii). Each section is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. Recognition that the medians or centers are almost the same for the three sites
2. Recognition that the variability (ranges, IQRs, spread) is different across the three sites
3. Context is included

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

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Question 4 (continued)

Notes:

- In all sections, comments about shape should be ignored because complete shape information is not obtainable from boxplots.
- Responses are not required to give numerical values. If responses provide numerical values, any reasonable approximation from the boxplots is acceptable.
- Because the boxplots are all symmetric, it is acceptable if the response discusses means instead of medians.
- Any discussion of chemical X and chemical Y is considered extraneous.
- Context is satisfied by any of the following references: site, chemical, weight, total weight, X, Y, or Z.

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. Site III is chosen.
2. Sums of the minimum and maximum are computed for the three chemicals at each site.
3. A reasonable numerical justification is given involving sums of a statistical measure across the three chemicals to choose Site III.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:

- If the response computes only the sum of the minimums for Site I and the sum of the maximums for Site II and recognizes that this is sufficient, the response is scored E.
- If an alternative measure is used that involves sums of the three chemicals, such as the sum of the medians or the sums of the first quartiles and sums of the third quartiles, instead of the minimum and maximum sums, the second component is not satisfied, but the third component might be satisfied.
    - If the response explicitly or implicitly compares the alternate sum to the other two sites (for example, by indicating that the sum is the closest to 20.5 percent or by listing the sums for all three sites) the response is scored P.
    - If the response does not have an implicit or explicit comparison, the response is scored I.
- If either Site I or Site II is identified as the correct choice, no matter how that choice is justified, the response is scored I.
- The approximate sums of the medians are 27.5 for Site I, 16 for Site II, and 20 for Site III.

Section 3 is scored as follows:

Essentially correct (E) if the response chooses chemical Y *AND* gives a reasonable justification based on the fact that the distributions of chemical Y are distinctive across sites.

Partially correct (P) if the response chooses chemical Y *AND* provides justification based on the boxplots, but does not clearly explain that the distributions of chemical Y are distinctive across sites;
OR
if the response correctly discusses that the distributions of chemical Y are distinctive across sites, but never explicitly chooses chemical Y as the best choice, for instance, by stating only that there is substantial overlap across sites for chemicals X and Z but no overlap for chemical Y.

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Question 4 (continued)

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- To justify that the distributions of chemical Y are distinctive across sites, the justification must address both location and variability of the boxplots; for example, by stating that the boxplots do not overlap for chemical Y.
- If the response chooses chemical X or Z *OR* chooses chemical Y with no reasonable justification, the response is scored I.
- The justification that the distributions of chemical Y are distinctive across sites:
  - The following are acceptable because both location <u>and</u> variability are addressed. Such responses are scored E.
    - The boxplots for chemical Y do not overlap, or the boxplots for chemicals X and Z overlap.
    - All values of Site I are high, all values of Site II are low, and all values of Site III are in the middle.
    - The ranges never intersect.
    - The boxplots share no data.
    - Has completely different percentages at each site.
  - The following are incomplete justifications and are scored P.
    - The boxplots vary.
    - Chemical Y varies the most.
    - Chemical Y has the greatest variation.
    - The variation between/among sites is the largest.
    - The boxplots are different.
    - The medians/means differ.
    - The medians/means are most variable.
    - There is a difference in the percentages of chemical Y for each site.
    - The distribution of percents differs the most among the sites.

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Question 4 (continued)

4 Complete Response

Three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No sections essentially correct and two sections partially correct

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Question 5

Intent of Question

The primary goal of this question was to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess a student's ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and p-value; and (4) draw an appropriate conclusion, with justification, in the context of the study.

Solution

Step 1: State a correct pair of hypotheses.

The null hypothesis is that age group at diagnosis and gender are independent (that is, they are not associated) for the population of people currently being treated for schizophrenia.

The alternative hypothesis is that age group at diagnosis and gender are not independent for the population of people currently being treated for schizophrenia.

Step 2: Identify a correct test procedure (by name or formula) and check appropriate conditions.

The appropriate test is a chi-square test of independence.

The conditions for this test are satisfied because:

1. The question states that the sample was randomly selected.
2. The expected counts for the eight cells of the table are at least 5, as seen in the following table, with expected counts shown below observed counts.

	Age at Diagnosis				
	20 to 29	30 to 39	40 to 49	50 to 59	Total
Women	46	40	21	12	119
	56.91	36.22	17.25	8.62	
Men	53	23	9	3	88
	42.09	26.78	12.75	6.38	

Step 3: Find the value of the test statistic and the p-value.

The test statistic is calculated as $\chi^2 = \sum \frac{(O-E)^2}{E}$, or

$$\begin{aligned}\chi^2 &= 2.093 + 0.395 + 0.817 + 1.322 \\ &\quad + 2.830 + 0.534 + 1.105 + 1.788 \\ &= 10.884.\end{aligned}$$

The p-value is $P(\chi^2 \geq 10.884) = 0.012$, based on $(4-1) \times (2-1) = 3$ degrees of freedom.

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Question 5 (continued)

Step 4: State the conclusion in context, with linkage to the p-value.

Because the p-value is very small (for instance much smaller than $\alpha = 0.05$), we reject the null hypothesis and conclude that the sample data provide strong evidence that there is an association between age group at diagnosis and gender for the population currently being treated for schizophrenia.

Scoring

This question is scored in three sections. Section 1 consists of steps 1 and 2 (stating the correct hypotheses, identifying the appropriate test procedure, and checking the technical conditions); section 2 consists of step 3 (performing the correct mechanics); and section 3 consists of step 4 (stating a correct conclusion with justification). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:
1. States BOTH hypotheses correctly with context included in at least one of them
2. Identifies a chi-square test of independence by name or formula
3. Verifies appropriate conditions that minimally include the condition for the expected counts and do not include any incorrect conditions (such as normality)

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:
- Stating the expected count condition is not sufficient; the condition must be checked by reporting the expected counts, or minimally by showing that the smallest expected count is at least 5.
- The random sample condition was stated in the stem so need not be explicitly checked.
- If the null and alternative hypothesis are correctly stated in terms of population proportions, component 1 is satisfied. For example:

  $H_0 : p_1 = p_2 = p_3 = p_4$, where p_i is the population proportion of women at each indicated age group, 1, 2, 3, or 4, who are currently being treated for schizophrenia.

  H_a : At least one of the population proportions, p_1, p_2, p_3, p_4, differs from the other three.

  OR

  H_a : The population proportions for the four age groups are not all the same.

Section 2 is scored as follows.

Essentially correct (E) if the response correctly calculates the following two values:
1. The value of the chi-square test statistic
2. The p-value, critical value, or p-value range from chi-square table

Partially correct (P) if the response correctly calculates only one of the two values.

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Question 5 (continued)

Incorrect (I) if the response does not satisfy the criteria for E or P.

Notes:

- If the response makes an error in one calculation, subsequent calculations are considered correct if they follow correctly from the initial miscalculation.
- With 3 degrees of freedom, the correct critical value is 7.81 for a significance level of 0.05 and 11.34 for a significance level of 0.01.
- Work does not have to be shown for calculations of test statistic or p-value. However, if incorrect work (other than minor arithmetic/transcription errors) is shown it is considered to be an incorrect calculation of the respective component, even if the correct value is given.
- If a response provides a test statistic that is not a chi-square test statistic, section 2 is scored I.

Section 3 is scored as follows.

Essentially correct (E) if the response includes the following three components:

1. A correct conclusion about the alternative hypothesis.
2. Justification of the conclusion based on linkage between the p-value and a reasonable alpha (or linkage between test statistic and critical value).
3. The conclusion is stated in context.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:

- If the response provides a correct decision, in context, with linkage to the p-value, but the decision is stated in terms of the null hypothesis with no conclusion about the alternative hypothesis, component 1 is not satisfied.
- Incorrect statistical statements are considered incorrect conclusions for the hypothesis test and do not satisfy component 1.
- If the conclusion is consistent with the p-value from section 2, and also in context with justification based on linkage to the p-value, section 3 is scored E.
- If no alpha level is given, the solution must be explicit about the linkage by giving a correct interpretation of the p-value or explaining how the conclusion follows from the p-value. For example, stating that because the p-value is small, we reject the null hypothesis or stating that because the p-value is large, we do not reject the null hypothesis.
- A decision about the null hypothesis (reject H_0 or fail to reject H_0) is not required, but if such a statement is given the scoring of the decision is considered in component 2.

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Question 5 (continued)

4 Complete Response

Three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct

OR

No sections essentially correct and two sections partially correct

OR

Section 1 partially correct and the other two sections incorrect

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Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate probabilities associated with treatment and control group memberships for two different methods of random assignment and (2) justify which method of random assignment is more appropriate in a given situation.

Solution

Part (a):

(i) Let T (tail) represent being assigned to the treatment group and H (head) represent being assigned to the control group for each coin flip. The process stops when either the treatment group or the control group has two members. The outcomes and their probabilities are as follows.

Arrangement	A	B	C	D	E	F
Chip outcomes	TT	THT	THH	HH	HTH	HTT
Calculation	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$	$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$
Probability	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(ii) Man 1 and Man 2 are assigned to the same group for arrangements A and D, so the probability is

$$P(\text{A}) + P(\text{D}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

Part (b):

(i) Let T represent being assigned to the treatment group and C represent being assigned to the control group for each chip drawn. The process stops when either the treatment group or the control group has two members. The probabilities differ from the coin flip method because chips are drawn *without* replacement. The outcomes and their probabilities are as follows.

Arrangement	A	B	C	D	E	F
Chip outcomes	TT	TCT	TCC	CC	CTC	CTT
Calculation	$\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)$	$\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$	$\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$	$\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)$	$\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$	$\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)$
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(ii) Man 1 and Man 2 are assigned to the same group for arrangements A and D, so the probability is

$$P(\text{A}) + P(\text{D}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}.$$

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Question 6 (continued)

Part (c):

Use the chip method. The chip method gives equal probability to all possible arrangements, but the coin method does not, as shown in the tables from parts (a-i) and (b-i). Furthermore, the coin method is more likely to result in imbalanced treatment groups with regard to students and teachers, based on the probabilities in parts (a-ii) and (b-ii). If food preferences for teachers are different than for students, the imbalance is a problem. For example, if one treatment group consists entirely of students, it would be impossible to know if a difference in the response variable is due to the treatment (type of meal) or the role of the person at the school (teacher or student).

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. The correct probability for each arrangement in the table of (i).
2. Appropriate justification is shown in (i).
3. The correct probability is given in (ii).

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- For component 2, examples of appropriate justification include the following:
  - Shows calculation of a correct probability for at least one of arrangements A and D *AND* shows calculation of a correct probability for at least one of arrangements B, C, E, and F. Calculations must be linked to a specific arrangement with a label, by proximity to an arrangement in the table, or by following the structure of the table or display of arrangements.
  - Lists correct coin outcomes for at least one of arrangements A and D *AND* lists correct coin outcomes for at least one of arrangements B, C, E, and F (for example, A: TT and B: THT). Lists of outcomes must be linked to a specific arrangement with a label, by proximity to an arrangement in the table, or by following the structure of the table or display of arrangements.
  - Shows a tree diagram that ends with six branches corresponding to the six arrangements. Probabilities or coin outcomes must be listed on the branches, but arrangements do not need to be identified.
  - Shows a tree diagram that ends with eight (or 16) branches where the branches leading to at least one of arrangements A and D are identified, and the branches leading to at least one of arrangements B, C, E, and F are identified. Probabilities or coin outcomes must be listed on the branches.
- If an incorrect probability is given in part (ii), component 3 is satisfied if the probability is between 0 and 1 and:
  - is the result of a minor arithmetic error; *OR*
  - is the result of adding the probabilities of arrangements A and D from the table in (i).

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Question 6 (continued)

Part (b) is scored as follows:

Essentially correct (E) if the response includes the following two components:
1. The correct probability for each arrangement in the table of (i).
2. The correct probability is given in (ii).

Partially correct (P) if the response includes only one of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: If an incorrect probability is given in (ii), component 2 is satisfied if the probability is between 0 and 1 and

- is the result of a minor arithmetic error; *OR*
- is the result of adding the probabilities of arrangements A and D from the table in (i); *OR*
- work is shown that demonstrates the same incorrect approach is used in both parts (a-ii) and (b-ii). For example, component 2 is satisfied if the response forgets to include $P(\mathrm{D})$ and says $P(\mathrm{A}) = \frac{1}{4}$ in (a-ii) and $P(\mathrm{A}) = \frac{1}{6}$ in part (b-ii). However, unlabeled answers of $\frac{1}{4}$ in (a-ii) and $\frac{1}{6}$ in (b-ii) would not satisfy component 2 because it is unclear if the same approach is being used.

Part (c) is scored as follows:

Essentially correct (E) if the response chooses the chip method AND includes the following three components:
1. Provides a statistical benefit to the chip method. For example, stating that all arrangements are equally likely.
2. Provides a statistical drawback to the coin method. For example, stating that the coin method is more likely to result in imbalanced treatment groups.
3. States that the responses (opinions, food preferences) of teachers and students might be different.

Partially correct if the response chooses the chip method and includes only one or two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:
- A correct comparative statement can be used to satisfy both components 1 and 2. For example, stating that the coin method is *more likely* to result in a disproportionate number of students in one treatment group, or that the chip method has a *greater chance* of providing roughly equivalent treatment groups.
- Benefits and drawbacks must be about the probabilities of the arrangements (from the tables in (i)), or the probabilities of imbalanced (balanced) groups (from the questions in (ii)), or both.
- If the response chooses the coin method, look back to see if the coin method should be the preferred method based on incorrect work from parts (a) and (b). If so, score part (c) using the corresponding three components.

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Question 6 (continued)

- If the response says that it does not matter which method to use, look back to see if the tables are the same in parts (a-i) and (b-i) or if the probabilities are the same in parts (a-ii) and (b-ii). If so, part (c) is partially correct if the response justifies the decision by stating that the two tables or the two probabilities are the same. To be essentially correct, the response also needs to satisfy component 3.
- If the response does not choose a method, deduct one component from the number of correct components.

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

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